

Finite-time Stability of Heating Furnace Temperature Control System

GUANG Xiaona, LIU Xiaohua

(Ludong University, Yantai 264039, China)

Abstract: A finite-time stabilization controller for the heating furnace temperature control system is proposed. Based on the extended Lyapunov finite-time stability theory and power integral method, a finite-time stable condition of the heating furnace temperature control system is given. The temperature of the heating furnace is directed by the finite-time stabilization controller to make it stable in finite time. And the quality and quantity of slabs is improved. The simulation example is presented to illustrate the applicability of the developed results.

Key words: Finite-time Stability, Heating Furnace, Temperature Control System, Power Integral Control, Lyapunov Function

1 Introduction

Slabs heating in a heating furnace is a typical industrial process [1]. Since the heated slabs that meet the production process requirements are controlled by the heating furnace temperature control system, the heating furnace temperature control system plays an important role in the process of slabs heating. Due to the function of the heating furnace temperature control system, many control methods on heating furnace temperature control system have appeared in the literature, such as neural network Smith predictive control [2], fuzzy PID control [3], fuzzy neural network control [4], fuzzy control [5], PID control [6] and so on. However, the stability control issue has not been concerned. In literature [7-8], the asymptotically stabilization of the heating furnace temperature control system is studied. It is obvious that the traditional stability only can describe the asymptotic behaviour of the system as time goes to infinity. Nevertheless, the stability fails when people want to know the behaviour of the system in finite time. As a consequence, finite-time stability has become popular in recent years.

As for general system, finite-time stabilization

controller is constructed in literature [9] by the power integral method [10-11]. Literature [12] proposes a power integral finite-time stabilization controller for the second-order system subject to unknown bounded disturbance. Power integral finite-time stabilization controller for high class connected systems is developed in literature [13-14]. Finite-time stabilization controller for P-normal-class high-order nonlinear systems is constructed in literature [15] by the power integral method. Motivated by the characteristics of power integral finite-time stabilization controller, the power integral method in the establish of finite-time stabilization controllers is also applied to some practical systems, such as aerospace attitude control [16-17], missile pilot [18], surface vehicle control [19] and so on. However, the power integral finite-time stabilization controller of the heating furnace temperature control system has not been reported.

The slabs are heated while transported through the furnace in steps. The temperature of the slabs is controlled by varying the zone temperature. Under the finite-time stabilization controller, the heating furnace temperature control system states can be stabilized to the origin in a finite time and all slabs reach their de-

sired final temperature range in finite-time. Therefore, the fuel is saved and the slab quality is improved. This paper introduced a finite-time power integral controller and finite-time stability condition of heating furnace temperature system. Different from the power integral finite-time stability controller raised in reference [9], the extended Lyapunov finite-time stability theorem [20] is employed to establish the finite-time stability. The influence of time constant, gain and time-delay on the finite-time stability of the heating furnace temperature control system is given through simulation.

The rest of this article is organized as follows. In Section 2, some preliminary results are presented. In Section 3, stability of heating furnace temperature control system is analyzed. In Section 4, numerical examples are used to illustrate the effectiveness of the theoretical results and the influence of parameter changes on the finite-time stability of the heating furnace temperature control system. Finally, in Section 5, general remarks are made for conclusion.

2 Preliminaries

Lemma 1^[21]. Considering the following system

$$\dot{x} = f(x(t)), f(0) = 0, x \in R^n, x(0) = x_0$$

Where, $f: D \rightarrow R^n$ is continuous on an open neighborhood D of the origin $x = 0$.

The zero solution of the system is finite-time convergent if there is an open neighborhood $U \in D$ of the origin and a function $T: U \setminus \{0\} \rightarrow (0, \infty)$, such that $\forall x_0 \in U$, the solution $\Psi(t, x_0)$ of the system is defined and $\Psi(t, x_0) \in U \setminus \{0\}$ for $t \in [0, T(x_0))$, and

$\lim_{t \rightarrow T(x_0)} \Psi(t, x_0) = 0$. Then, $T(x_0)$ is called the settling time. If the zero solution of the system is finite-time convergent, the set of point x_0 such that $\Psi(t, x_0) \rightarrow 0$ is called the domain of attraction of the solution. The zero solution of the system is finite-time stable if it is Lyapunov stable and finite-time convergent. When, $U = D = R^n$, the zero solution is said to

be globally finite-time stable.

Lemma 2^[3]. If, $0 < p = \frac{p_1}{p_2} < 1$, where, $p_1 > 0$,

$p_2 > 0$ is positive odd number, then

$$x^p - y^p \leq 2^{1-p} |x - y|^p$$

Lemma 3^[8]. For, $x \in R, y \in R$, let c, d, γ be a positive real number, then

$$|x|^c |y|^d \leq \frac{c}{c+d} \gamma |x|^{c+d} + \frac{d}{c+d} \gamma^{-\frac{c}{d}} |y|^{c+d}$$

Lemma 4^[3]. For, $x_i \in R, i = 1, \dots, n, 0 < p \leq 1$ be a positive real number, then

$$(|x_1| + |x_2| + \dots + |x_n|)^p \leq |x_1|^p + |x_2|^p + \dots + |x_n|^p$$

Lemma 5^[20]. system

$$\dot{x}(t) = f(x(t)), f(0) = 0, t_0 = 0, x_0 \triangleq x(0)$$

Where, $x(t) \in R^n$ represent the state, $f: U \rightarrow$

R^n is a continuous function in the domain U containing the origin to the dimensional space $R^n, 0 \in R^n$ represents the zero vector, x_0 represents the initial state. If there is a positive definite and continuous function $V(x)$ in the defined domain U , it satisfies

$$\dot{V}(x) \leq -\alpha V(x)^p + \eta, \forall x \in U$$

Where, $\alpha > 0, 0 < \eta < \infty, p \in (0, 1)$.

Then the system is actually stable in finite time. The convergence time of the system state is as follows:

$$T \leq \frac{V^{1-p}(x_0)}{p\theta_0(1-\alpha)}$$

Where, $V(x_0)$ is the initial value of $V(x)$.

3 Finite-time Stability Analysis of Heating Furnace Temperature Control System

General industrial objects can be represented by a first-order system or second-order system^[22]. Heating furnace temperature control system is represented by a second-order system in literature [23], which can be expressed in the form of the following transfer function

$$G(s) = \frac{Ke^{-\tau s}}{(s+1)(Ts+1)} \quad (1)$$

Where, K represents the gain, T represents the time constant, τ represents the time-delay.

Without considering the pure time-delay $e^{-\tau s}$, according to the method in literature [24], the transfer function is transformed into the following state-space form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - (1+T)x_2 + u \end{aligned} \quad (2)$$

The sufficient condition under which the system (2) can be stabilized in finite time by a continuous state feedback are as follows

$$-x_1 - (1+T)x_2 \leq 0 \quad (3)$$

The finite-time stabilization controller is constructed as follows

$$\begin{aligned} u &= -\left(2 - \frac{1}{m}\right) 2^{\frac{1}{q}} k_1^{1+m} k_2 (x_2^m + k_1^m x_1)^{\frac{2}{m}-1} \\ &+ x_1 + (1+T)x_2 \end{aligned} \quad (4)$$

Where, $k_1 > 0, k_2 \geq 0, 1 < m < \frac{m_1}{m_2} < 2, m_1, m_2$ is positive odd.

Theorem 1. Let the heating furnace temperature control system is represented by Eq. (2). If the finite-time stabilization controller is set as Eq. (4), under the finite-time stabilization sufficient condition Eq. (3), the system Eq. (2) can be stabilized in finite time.

Proof: First, select a lyapunov function

$$V_1(x_1) = \frac{1}{2} x_1^2 \quad (5)$$

According to system Eq. (2), we have

$$\dot{V}_1(x_1) = x_1 x_2 \leq x_1 (x_2 - x_2^*) + x_1 x_2^* \quad (6)$$

Where, x_2^* is virtual controller.

Then the continuous virtual controller x_2^* can be defined as

$$x_2^* = -k_1 x_1^{\frac{1}{m}} \quad (7)$$

Where, $k_1 > 0, 1 < m < \frac{m_1}{m_2} < 2, m_1, m_2$ is positive odd.

Put virtual controller Eq. (7) into Eq. (6), we have

$$\dot{V}_1(x_1) \leq -k_1 |x_1|^{1+\frac{1}{m}} + x_1 |x_2 - x_2^*| \quad (8)$$

According to lemma 2, we have

$$\begin{aligned} |x_2 - x_2^*| &= \left| (x_2^m)^{\frac{1}{m}} - (x_2^{*m})^{\frac{1}{m}} \right| \\ &\leq 2^{\frac{1}{m}} |x_2^m - x_2^{*m}|^{\frac{1}{m}} \leq 2^{\frac{1}{m}} |\xi|^{\frac{1}{m}} \end{aligned} \quad (9)$$

Where, ξ is denoted as: $\xi = x_2^m - x_2^{*m}$.

Substituting Eq. (9) into Eq. (8), we have

$$\dot{V}_1(x_1) \leq -k_1 |x_1|^{1+\frac{1}{m}} + 2^{\frac{1}{m}} |x_1| |\xi|^{\frac{1}{m}} \quad (10)$$

According to lemma 3, we have

$$\begin{aligned} 2^{\frac{1}{m}} |x_1| |\xi|^{\frac{1}{m}} &\leq 2^{\frac{1}{m}} \left[\frac{1^* c_1}{1 + \frac{1}{m}} |x_1|^{1+\frac{1}{m}} + \frac{\frac{1}{m} c_1}{1 + \frac{1}{m}} |\xi|^{1+\frac{1}{m}} \right] \\ &\leq 2^{\frac{1}{m}} \left[\frac{m c_1}{1+m} |x_1|^{1+\frac{1}{m}} + \frac{c_1^{-m}}{1+m} |\xi|^{1+\frac{1}{m}} \right] \\ &= 2^{\frac{1}{m}} \left[m c_1 (1+m)^{-1} |x_1|^{1+\frac{1}{m}} + c_1^{-m} (1+m)^{-1} |\xi|^{1+\frac{1}{m}} \right] \end{aligned} \quad (11)$$

Where, $0 < c_1 = 2^{-1+\frac{1}{m}} k_1 \theta_1 m^{-1} (1+m), 0 < \theta_1 < \frac{1}{2}$.

Substituting Eq. (11) into Eq. (10), we have

$$\begin{aligned} \dot{V}_1(x_1) &\leq -k_1 |x_1|^{1+\frac{1}{m}} + 2^{\frac{1}{m}} |x_1| |\xi|^{\frac{1}{m}} \\ &\leq -k_1 |x_1|^{1+\frac{1}{m}} + 2^{\frac{1}{m}} \left[m c_1 (1+m)^{-1} |x_1|^{1+\frac{1}{m}} \right] \\ &+ 2^{\frac{1}{m}} \left[c_1^{-m} (1+m)^{-1} |\xi|^{1+\frac{1}{m}} \right] \\ &= \left[-k_1 + 2^{\frac{1}{m}} m c_1 (1+m)^{-1} \right] |x_1|^{1+\frac{1}{m}} \\ &+ 2^{\frac{1}{m}} c_1^{-m} (1+m)^{-1} |\xi|^{1+\frac{1}{m}} \\ &= \left[-k_1 + 2^{\frac{1}{m}} m 2^{-1+\frac{1}{m}} k_1 \theta_1 m^{-1} (1+m) (1+m)^{-1} \right] \\ &|x_1|^{1+\frac{1}{m}} + 2^{\frac{1}{m}} c_1^{-m} (1+m)^{-1} |\xi|^{1+\frac{1}{m}} \\ &= -(1-\theta_1) k_1 |x_1|^{1+\frac{1}{m}} + 2^{\frac{1}{m}} c_1^{-m} (1+m)^{-1} |\xi|^{1+\frac{1}{m}} \end{aligned} \quad (12)$$

Consider the Lyapunov equation as follows

$$V_2(x_1, x_2) = V_1(x_1) + \frac{1}{\left(2 - \frac{1}{m}\right) 2^{\frac{1}{m}} k_1^{1+m}} \int_{x_2^*}^{x_2} (s^m - x_2^{*m})^{2-\frac{1}{m}} ds \quad (13)$$

It yields

$$\frac{\partial x_2^{*q}}{\partial x_1} = \frac{\partial \left(-k_1 x_1^m\right)}{\partial x_1} = \frac{\partial (-k_1)^m x_1}{\partial x_1} = -k_1^m \quad (14)$$

$$\int_{x_2^*}^{x_2} (s^m - x_2^{*m})^{2-\frac{1}{m}} ds \leq 2^{1-\frac{1}{m}} |\xi|^2 \quad (15)$$

To convert Eq. (15), we have

$$|\xi| \geq 2^{\frac{1}{2m}-\frac{1}{2}} \left(\int_{x_2^*}^{x_2} (s^m - x_2^{*m})^{2-\frac{1}{m}} ds \right)^{\frac{1}{2}} \quad (16)$$

Derivation of $V_2(x_1, x_2)$, we have

$$\begin{aligned} \dot{V}_2(x_1, x_2) &\leq -k_1 |x_1|^{1+\frac{1}{m}} + x_1 (x_2 - x_2^*) \\ &+ \frac{1}{2^{\frac{1}{m}} k_1} x_2 \int_{x_2^*}^{x_2} (s^m - x_2^{*m})^{1-\frac{1}{m}} ds \\ &+ \frac{1}{\left(2 - \frac{1}{m}\right) 2^{\frac{1}{m}} k_1^{1+m}} \xi^{2-\frac{1}{m}} (-x_1 - (1+T)x_2 + u) \quad (17) \end{aligned}$$

Substituting Eq. (9) and Eq. (15) into Eq. (17), we have

$$\begin{aligned} \dot{V}_2(x_1, x_2) &\leq -k_1 |x_1|^{1+\frac{1}{m}} + 2^{1-\frac{1}{m}} |x_1| |\xi|^{\frac{1}{m}} + \frac{1}{k_1} |x_2| |\xi| \\ &+ \frac{1}{\left(2 - \frac{1}{m}\right) 2^{\frac{1}{m}} k_1^{1+m}} \xi^{2-\frac{1}{m}} (-x_1 - (1+T)x_2 + u) \quad (18) \end{aligned}$$

According to lemma 2, we have

$$\begin{aligned} |x_2| |\xi| &\leq |x_2 - x_2^* + x_2^*| |\xi| \leq |x_2 - x_2^*| |\xi| \\ &+ |x_2^*| |\xi| \leq 2^{1-\frac{1}{m}} |\xi|^{1+\frac{1}{m}} + k_1 |x_1|^{\frac{1}{m}} |\xi| \quad (19) \end{aligned}$$

According to lemma 3, we have

$$k_1 |x_1|^{\frac{1}{m}} |\xi| \leq k_1 \left[\frac{1^* c_2}{1 + \frac{1}{m}} |x_1|^{1+\frac{1}{m}} + \frac{\frac{1}{m} c_1}{1 + \frac{1}{m}} |\xi|^{1+\frac{1}{m}} \right]$$

$$\leq k_1 \left[\frac{m c_2}{1+m} |x_1|^{1+\frac{1}{m}} + \frac{c_2^{-m-1}}{1+m} |\xi|^{1+\frac{1}{m}} \right] \quad (20)$$

$$= m c_2 k_1 (1+m)^{-1} |x_1|^{1+\frac{1}{m}} + c_2^{-m-1} k_1 (1+m)^{-1} |\xi|^{1+\frac{1}{m}}$$

Where, $0 < c_2 = k_1 \theta_2 m^{-1} (1+m)$, $0 < \theta_2 < \frac{1}{2}$.

Substituting Eq. (20) into Eq. (19), we have

$$\begin{aligned} |x_2| |\xi| &\leq 2^{1-\frac{1}{m}} |\xi|^{1+\frac{1}{m}} + m c_2 k_1 (1+m)^{-1} |x_1|^{1+\frac{1}{m}} \\ &+ c_2^{-m-1} k_1 (1+m)^{-1} |\xi|^{1+\frac{1}{m}} \quad (21) \end{aligned}$$

Substituting Eq. (21) into Eq. (18), we have

$$\dot{V}_2(x_1, x_2) \leq -k_1 |x_1|^{1+\frac{1}{m}} + 2^{1-\frac{1}{m}} |x_1| |\xi|^{\frac{1}{m}} + \frac{1}{k_1} 2^{1-\frac{1}{m}} |\xi|^{1+\frac{1}{m}}$$

$$+ m c_2 (1+m)^{-1} |x_1|^{1+\frac{1}{m}} + c_2^{-m-1} (1+m)^{-1} |\xi|^{1+\frac{1}{m}}$$

$$+ \frac{1}{\left(2 - \frac{1}{m}\right) 2^{\frac{1}{m}} k_1^{1+m}} \xi^{2-\frac{1}{m}} (-x_1 - (1+T)x_2 + u)$$

$$\dot{V}_2(x_1, x_2) \leq -k_1 |x_1|^{1+\frac{1}{m}} + 2^{1-\frac{1}{m}} |x_1| |\xi|^{\frac{1}{m}} + k_1 \theta_2 |x_1|^{1+\frac{1}{m}}$$

$$+ a_1 |\xi|^{1+\frac{1}{m}} + \frac{1}{\left(2 - \frac{1}{m}\right) 2^{\frac{1}{m}} k_1^{1+m}} \xi^{2-\frac{1}{m}} \quad (22)$$

$$(-x_1 - (1+T)x_2 + u)$$

Where, $a_1 = \frac{1}{k_1} 2^{1-\frac{1}{m}} + c_2^{-m-1} (1+m)^{-1}$.

Substituting Eq. (12) into Eq. (22), we have

$$\dot{V}_2(x_1, x_2) \leq -(1-\theta_1) k_1 |x_1|^{1+\frac{1}{m}}$$

$$+ 2^{1-\frac{1}{m}} c_1^{-m} (1+m)^{-1} |\xi|^{1+\frac{1}{m}} + k_1 \theta_2 |x_1|^{1+\frac{1}{m}} + a_1 |\xi|^{1+\frac{1}{m}}$$

$$+ \frac{1}{\left(2 - \frac{1}{m}\right) 2^{\frac{1}{m}} k_1^{1+m}} \xi^{2-\frac{1}{m}} (-x_1 - (1+T)x_2 + u) \quad (23)$$

$$\leq -(1-\theta_1 - \theta_2) k_1 |x_1|^{1+\frac{1}{m}} + a_2 |\xi|^{1+\frac{1}{m}}$$

$$+ \frac{1}{\left(2 - \frac{1}{m}\right) 2^{\frac{1}{m}} k_1^{1+m}} \xi^{2-\frac{1}{m}} (-x_1 - (1+T)x_2 + u)$$

Where, $a_2 = 2^{1-\frac{1}{m}} c_1^{-m} (1+m)^{-1} + a_1$.

When u is substituted in, we have

$$\begin{aligned} \dot{V}_2(x_1, x_2) \leq & -(1-\theta_1-\theta_2)k_1|x_1|^{1+\frac{1}{m}} \\ & + a_2|\xi|^{1+\frac{1}{m}} - k_2\xi^{1+\frac{1}{m}} \end{aligned} \quad (24)$$

Substituting Eq. (5) and Eq. (16) into Eq. (24), we have

$$\begin{aligned} \dot{V}_2(x_1, x_2) \leq & -(1-\theta_1-\theta_2)k_1 2^{\frac{1}{2}+\frac{1}{2m}} V_1^{\frac{1}{2}+\frac{1}{2m}} \\ & - k_2 2^{\frac{1}{2m}-\frac{1}{2}} \left(\int_{x_2^*}^{x_2} (s^m - x_2^{*m})^{2-\frac{1}{m}} ds \right)^{\frac{1}{2m+\frac{1}{2}}} + a_2|\xi|^{1+\frac{1}{m}} \end{aligned} \quad (25)$$

Take $\alpha = \min \left\{ (1-\theta_1-\theta_2)k_1 2^{\frac{1}{2}+\frac{1}{2m}}, k_2 2^{\frac{1}{2m}-\frac{1}{2}} \right\}$,

we have

$$\dot{V}_2(x_1, x_2) \leq -\alpha V_2^{\frac{1}{2}+\frac{1}{2m}} + \eta \quad (26)$$

In other words, the system (2) is stable in actual finite time under controller u . Similarly, according to literature [24], the system (1) is also stable in finite time.

According to lemma 5, the convergence time satisfies

$$T \leq \frac{V_2^{\frac{1}{2}+\frac{1}{2m}}(x_0)}{\left(\frac{1}{2}-\frac{1}{2m}\right)\theta_0(1-\alpha)} \quad (27)$$

Where, $0 < \theta_0 < 1$.

4 Simulation

In this section, the simulation results of the system (2) are presented, which are under the action of the controller and the influence of parameters on finite

time stability of heating furnace temperature control system.

4.1 Simulation Example

The simulation results of the system (2) under the action of the controller are presented. The initial value is $x_1(0) = 1.5, x_2(0) = -3$, and the time constant is selected as $T = 1$, and the parameters of the power-integral controller is selected as $m = \frac{3}{2}, k_1 = 4, k_2 = 5$. At this point, the state response in example is shown in Fig.1.

As shown in the state response curve above, the system (1) can achieve finite-time stability under the action of the finite-time plus power integral controller.

4.2 Influence of Parameters on Finite-time Stability of Heating Furnace Temperature Control System

For the system (1), the simulation verifies the influence of time constant T , gain K and time-delay τ on finite-time stability of the system. Fixed gain K and time-delay τ , the state response at $T = 0.1, T = 0.5, T = 2, T = 5$ is shown in Fig.2-Fig.5. Fixed time constant T and time-delay τ , the state response at $K = 2, K = 5$ is shown in Fig.6 and Fig.7. Fixed time constant T and gain K , the state response at $\tau = 0.1, \tau = 0.2$ is shown in Fig.8 and Fig.9.

It can be seen from Fig.2-Fig.5 that with the increase of T , the convergence time of the system also increases. On the contrary, as T decreases, the convergence time of the system decreases. Meanwhile,

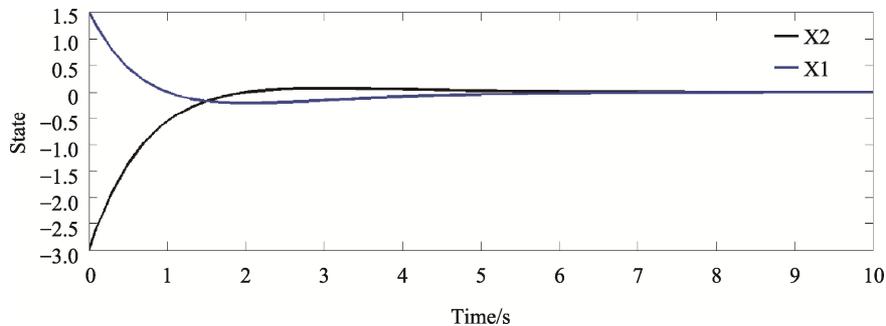
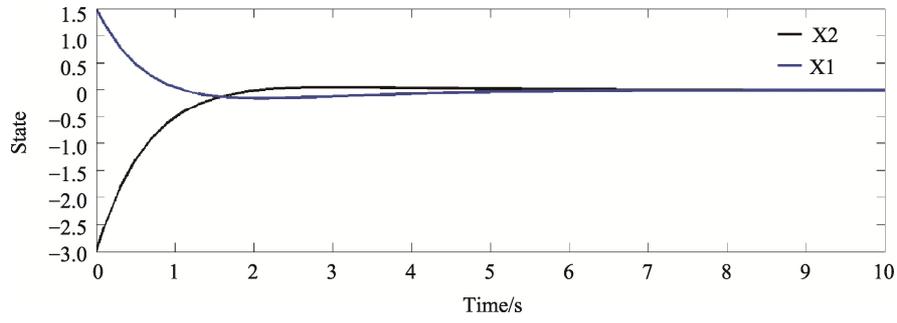
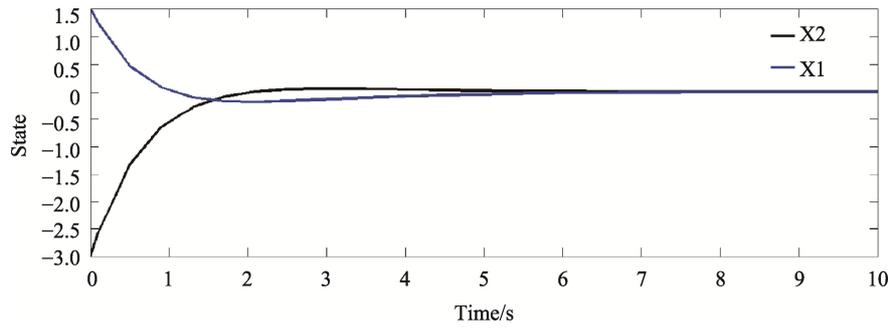
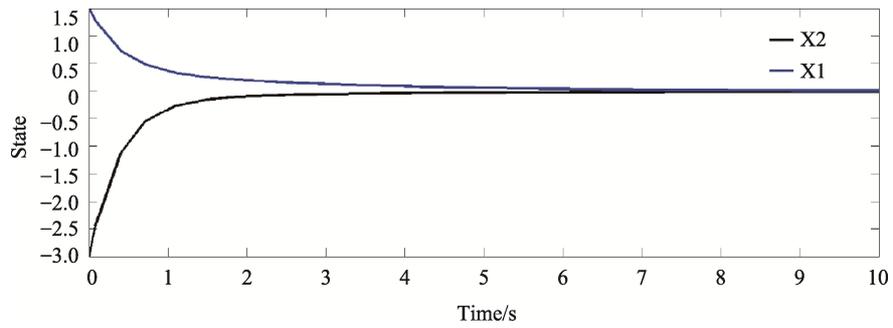
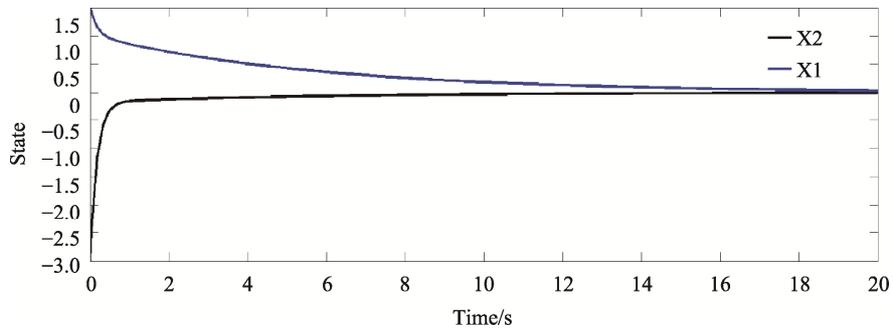


Fig.1 The State Response in Example

**Fig.2 The State Response at $T=0.1$** **Fig.3 The State Response at $T=0.5$** **Fig.4 The State Response at $T=2$** **Fig.5 The State Response at $T=5$**

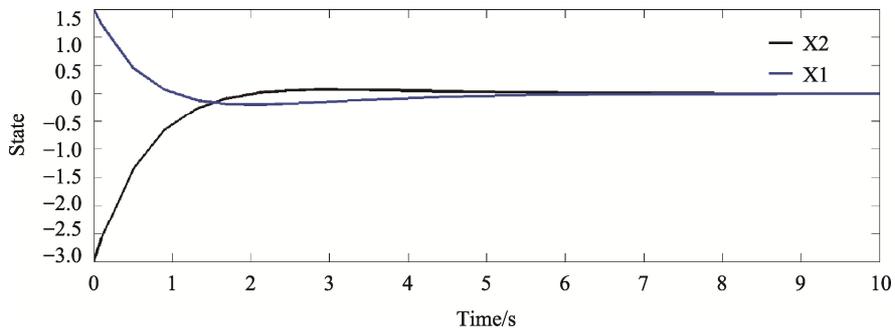


Fig.6 The State Response at $K=2$

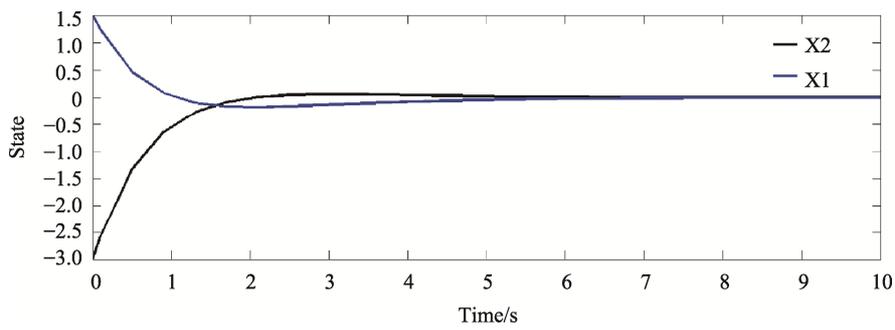


Fig.7 The State Response at $K=5$

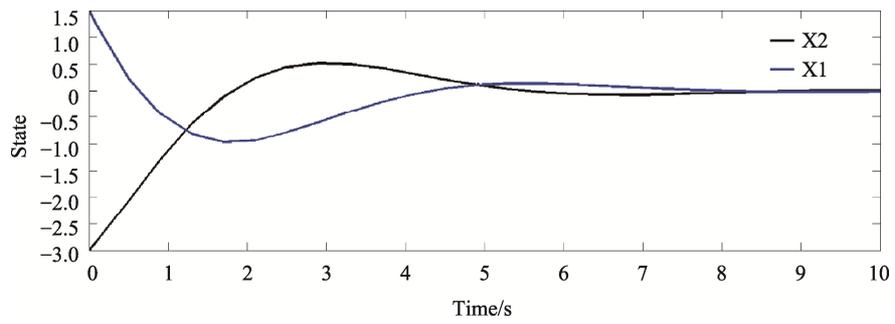


Fig.8 The State Response at $\tau=0.1$

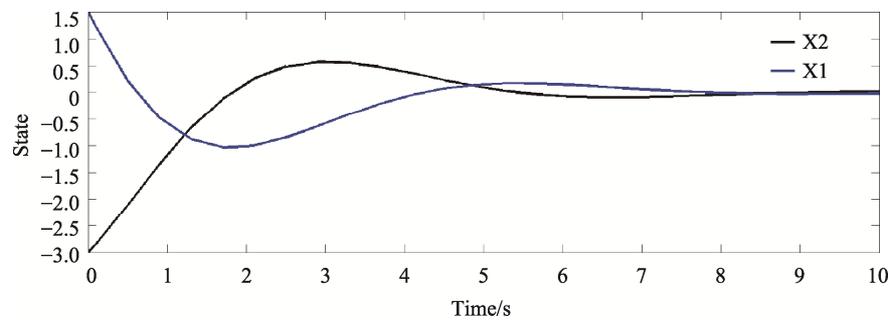


Fig.9 The State Response at $\tau=0.2$

with the increase of T , the delay-time of the system decreases. On the contrary, with the decrease of T , the delay-time of the system increases. It can be seen from the state response curve that the smaller the T is, the larger the overshoot of the system state is. Therefore, the time constant T has an impact on the finite-time stability of the system. According to Fig.6 and Fig.7, as gain K increases, the stable time increases but not significantly. In accordance with Fig.8 and Fig.9, the larger the time-delay τ is, the larger the overshoot of the system state is. In conclusion, appropriate parameters need to be set according to different requirements in the actual industrial process.

5 Conclusion

In this paper, motivated by the extended Lyapunov finite-time stability theorem and the power integral control method, the power integral finite-time stabilization controller is obtained. Sufficient condition for the finite-time stabilization of the heating furnace temperature control system is given. Simulation results show the effectiveness of the proposed method and the influence of time constant, gain and time-delay on the finite-time stabilization of the heating furnace temperature control system.

References

- [1] Acharya, A D. (1996). Reheat Furnace Temperature Control and performance at Essar Steel. *Iron and Sice-Enginur*, pp.139-149.
- [2] Liu, P. (1998). Predictive Control for Nonlinear System Using Neural Networks. *INT J Control*, 71(6), pp.256-274.
- [3] Michel, J. (1999). Fuzzy Logic Control of Industrial Heat Treatment Furnaces. *IEEE*, pp.839-843.
- [4] Mesbah, S. (1999). Fuzzy Neural Control with Application to a Heating System. *IEEE*, pp.533-538.
- [5] Guan, X. (2002). Application of Fuzzy Control Theory to Direct Heating Furnace Control System. *4th World Congress on Intelligent Control and Automation*, pp.396-399.
- [6] Sondhi, S. (2014). Stability Testing and IMC Based Fractional Order PID Controller Design for Heating Furnace System. *IEEE*, pp.1-5.
- [7] Pedersen, L M. (1998). On the Reheat Furnace Control Problem. *American Control Conference*, pp.3811-3815.
- [8] Steinboeck, A. (2011). Feedback Tracking Control of Continuous Reheating Furnaces. *18th IFAC World Congress*, pp.11744-11749.
- [9] Qian, C. (2001). A Continuous Feedback Approach to Global Strong Stabilization of Nonlinear Systems. *IEEE Transactions on Automatic Control*, 46(7), pp.1061-1079.
- [10] Dorato, P. (1961). Short Time Stability in Linear Time-varying Systems. *Proceedings of the IRE International Convention Record Part 4*, pp.83-87.
- [11] Bhat, S. (1998). Continuous Finite-time Stabilization of the Translational and Rational Double Integrators. *IEEE*, 43(5), pp.678-682.
- [12] Qian, C. (2001). Non-Lipschitz Continuous Stabilizers for Nonlinear Systems with Uncontrollable Unstable Linearization. *Systems & Control Letters*, 42(3), pp.185-200.
- [13] Huang, X. (2005). Global Finite-Time Stabilization of a Class of Uncertain Nonlinear Systems. *Automatica*, 41(5), pp.881-888.
- [14] Ding, S. (2009). Stability Analysis for a Second-order Continuous Finite-time Control System Subject to a Disturbance. *Journal of Control Theory and Applications*, 7(3), pp.271-276.
- [15] Yang, J. (2017). Global Output Regulation for a Class of Lower Triangular Nonlinear Systems: A Feedback Domination Approach. *Automatica*, 76, pp.65-69.
- [16] Sun, Z Y. (2017). A New Approach to Fast Global Finite-time Stabilization of High-order Nonlinear System. *Automatica*, 81, pp.455-463.
- [17] Du, H. (2012). Finite-Time Attitude Stabilization for a Spacecraft Using Homogeneous Method. *Journal of Guidance, Control, and Dynamics*, 35(3), pp.740-748.
- [18] Du, H. (2011). Finite-Time Attitude Tracking Control of Spacecraft with Application to Attitude Synchronization.

- IEEE Transactions on Automatic Control*, 56(11), pp.2711-2717.
- [19] Sun, H. (2011). A Composite Control Scheme for 6DOF Spacecraft Formation Control. *Acta Astronautica*, 69(7-8), pp.595-611.
- [20] He, S. (2016). Robust Missile Autopilots with Finite-Time Convergence. *Asian Journal of Control*, 18(3), pp.1010-1019.
- [21] Bhat, S. (2000). Finite-time Stability of Continuous Autonomous Systems. *SIAM Journal of Control and Optimization*, 38(3), pp.751-766.
- [22] Bi, Q. (1999). Robust Identification of First-order Plus Dead-time Model from Step Response. *Control Engineering Practice*, 7(1), pp.71-77.
- [23] Wang, LX. (2001). Structured Neural Networks for Constrained Model Predictive Control. *Automatica*, 37(6), pp.1235-1243.
- [24] Zheng, D Z. (2002). *linear system theory. (Second Edition)*. Tsinghua University Press.

Author Biographies



finite-time stability

Email: 1783506585@qq.com

GUANG Xiaona, was born in Shandong, in 1995. She received Bachelor's degree from Ludong University in 2017. Now she is a graduated student of Ludong University. Her current research interests are control and optimization and fi-



their applications.

Email: xhliu@ldu.edu.cn

LIU Xiaohua, received PhD from operations research and cybernetics from Dalian University of Technology in 2003. Now he is a professor in the School of Mathematics and Statistics Science, Ludong University. His main research inter-



Copyright: © 2020 by the authors. This article is licensed under a Creative Commons Attribution 4.0 International License (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).